

20030130113



National Defence
Défense nationale

UNCLASSIFIED

DTIC FILE COPY

UNLIMITED
DISTRIBUTION

DRES

SUFFIELD MEMORANDUM

NO. 1225

AD-A198 357

AN INFORMATION-THEORETIC METHOD FOR THE INVERSION OF THE LIDAR EQUATION (U)

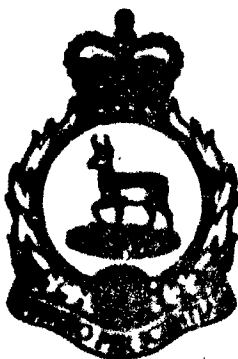
by

E. Yee

PCN 351SD

May 1968

DTIC
ELECTE
SEP 08 1968
S H D



Canada

DEFENCE RESEARCH | ESTABLISHMENT SUFFIELD, RALSTON, ALBERTA

Handwritten signature

DISTRIBUTION STATEMENT A

Approved for public release
Distribution Unlimited

88 9 8 034

UNCLASSIFIED
DEFENCE RESEARCH ESTABLISHMENT SUFFIELD
RALSTON ALBERTA

SUFFIELD MEMORANDUM NO. 1225

**AN INFORMATION-THEORETIC METHOD FOR THE INVERSION
OF THE LIDAR EQUATION (U)**

by

E. Yee

PCN 3519D

UNCLASSIFIED

UNCLASSIFIED

UNCLASSIFIED

DEFENCE RESEARCH ESTABLISHMENT SUFFIELD

RALSTON ALBERTA

SUFFIELD MEMORANDUM NO. 1225

AN INFORMATION-THEORETIC METHOD FOR THE INVERSION
OF THE LIDAR EQUATION (U)

by

E. Yee

ABSTRACT

A new solution is presented for the reconstruction of profiles of aerosol volume extinction coefficients from the noisy backscattered returns of a monostatic single-wavelength lidar system. This inverse problem is solved by utilising an information-theoretic method based on the principle of minimum cross-entropy (MCE), which represents an objective and rational approach for the effective incorporation, into the inversion procedure, of both prior information in the form of an initial estimate of the extinction coefficient and additional information in the form of the observed lidar data. A simple and robust numerical procedure, based on the ellipsoid algorithm, is developed to compute the MCE reconstruction of the extinction function. A number of numerical examples, based on noisy synthetic lidar data, are employed to demonstrate and evaluate the utility and efficacy of the inversion method.



UNCLASSIFIED

Accession For	
NTIS GRA&I	<input checked="checked" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

UNCLASSIFIED

DEFENCE RESEARCH ESTABLISHMENT SUFFIELD
RALSTON ALBERTA

SUFFIELD MEMORANDUM NO. 1225

AN INFORMATION-THEORETIC METHOD FOR THE INVERSION
OF THE LIDAR EQUATION (U)

by

E. Yee

INTRODUCTION

1. An optical remote sensing system based on the light detecting and ranging (lidar) principle can be utilised for the recovery of various properties (e.g., mass and number concentration, particle size distribution, and optical constants) of aerosol clouds from the backscattered return of a controlled laser pulse. Since present lidar systems, such as the Laser Cloud Mapper (LCM) [1], are capable of generating very short laser pulses at rapid repetition rates, these systems effectively possess the capability for rapid and detailed probing and surveillance of aerosol clouds with enhanced sensitivity and resolution. Indeed, the compactness, mobility and speed of operation afforded by the LCM makes it a valuable tool for the rapid monitoring, characterisation and assessment of threat aerosols which, as a consequence, can provide the user with a much earlier warning of potentially hazardous situations than can be provided by local monitoring procedures based on various direct in situ sampling methods. However, it is important to note that a direct (local) sensing technique provides a more comprehensive characterisation of the physical properties of the aerosols since it is based on the actual capture and examination of the particulates, whereas an optical remote sensing technique must infer these properties from the backscattered patterns of the electromagnetic waves.

2. Indeed, since atmospheric particulates interact with optical radiation in a complicated manner, the problem of inferring physical properties of atmospheric aerosols from backscattered radiation is not straightforward and is fraught with numerous difficulties. Consequently, there still does not exist a totally satisfactory solution to this problem, despite the considerable research effort expended on it over the past thirty years. The difficulty in formulating an appropriate solution to the lidar inverse problem stems from the fact that the reconstruction of the optical properties of an aerosol cloud from a limited set of noisy lidar return data is an inherently ill-posed problem in the sense that the solution is

UNCLASSIFIED

non-unique (viz., there are infinitely many extinction functions that are consistent with the given data) and unstable (viz., the solution does not depend continuously on the data). Consequently, the formal mathematical solution for the lidar inverse problem [2] was found to be plagued by instabilities and inaccuracies which effectively rendered such a solution useless for practical applications. In fact, the failure of this exact (i.e., formal) solution was not well understood and was originally attributed to the physical effects arising from the neglect of the multiple scattering contributions in the treatment rather than from the inherently ill-posed nature of the problem.

3. The lidar inverse problem can be uniquely solved provided one is given an ideal and complete set of lidar backscattering returns, but practical limitations imposed by the measurement system as well as some sweeping simplifications arising from the neglect of multiple scattering contributions, serve to thwart the attainment of the exact mathematical solution. It should be emphasized that under realistic circumstances, the exact solution cannot be used to completely characterize the true properties of the aerosol clouds from the observed noisy and discrete lidar data. Indeed, the exact solution does not even make any allowances for the noise in the data in any optimal manner. Nevertheless, all the solutions proposed for the inversion of the lidar equation to date have been based upon some appropriate modification of the exact mathematical solution. Klett [3] showed that the exact lidar inverse solution can be stabilized if one chooses the boundary value (i.e., the value of the extinction function at some specified range) at the far end of the aerosol cloud rather than at the near end. However, this required the determination of the boundary value which cannot be obtained from the data directly. Evans [4] demonstrated how the latter problem can be overcome by utilizing an appropriate calibration of the lidar system in conjunction with a simple modification of Klett's inversion procedure. Finally, it should be mentioned that two popular lidar inversion procedures, namely the slope and ratio methods [5], can be considered to be simply first-order approximations of the exact solutions.

4. In light of the numerous difficulties encountered in the formulation of an appropriate solution for the lidar inverse problem, it is important to reexamine the problem from the point of view of evaluating the physical validity and meaningfulness of the recovered solution in relation to the nature and quality of the data. As stated previously, although formal analytical solutions exist for the inversion of the lidar equation, the difficulties that arise in the implementation of these solutions reside in the fact that the backscattered radiation is not known precisely over the entire range interval. Indeed, the finite and noisy character inherent in all realistic lidar data ultimately contributes to the ill-posed nature of the lidar inverse problem. In this paper, an information-theoretic method is developed for the inversion of the lidar equation. This method permits the objective incorporation of prior information obtained from the physical character of the extinction function and of additional information encoded in the lidar data. The interaction between these two components of information is simple: prior knowledge relating to various physical characteristics that the solution must possess is utilized to eliminate a vast majority of members from the infinite set of solutions consistent with the finite uncertain data and from the

remaining candidates in this solution set, the principle of parsimony is utilized to select the simplest solution, i.e. the solution which possesses the least amount of detail or information that was not already known or expected. The basic philosophy behind seeking the simplest model (solution) resides in the fact that one does not want to be misled by any extraneous features in the solution that are not essential in matching the data. Alternatively, it is expected that the features which appear in the parsimonious solution are significant in that they are specifically required by the data. In this paper, the application of these ideas to the solution of the lidar inverse problem is quantified in the form of the principle of minimum cross-entropy.

THEORY

5. The single-scattering lidar equation which relates the volume extinction coefficient to the backscattered radiation for a monostatic, single-wavelength lidar system is given by

$$P(r) = P_0 \frac{cr}{2} \frac{\beta(r)A}{r^2} \exp \left\{ -2 \int_0^r \alpha_e(r') dr' \right\},$$

where r is the range, $P(r)$ is the backscattered power, P_0 is the initial laser pulse power, A is the receiver area, c is the velocity of light, r is the laser pulse time duration, $\beta(r)$ is the backscattering coefficient function, and $\alpha_e(r)$ is the volume extinction coefficient function. By normalising the lidar return power by the range and the system constants to form the derived signal $S(r) \equiv \ln[2r^2 P(r)/(cr P_0 A)]$, it is possible to rewrite the lidar equation in a form which removes the effects of geometric attenuation and pure system factors:

$$S(r) = \ln[\beta(r)] - 2 \int_0^r \alpha_e(r') dr'. \quad (1a)$$

6. In actual measurements, the observed backscattered intensity is corrupted by noise and, consequently, the measurement model is taken as

$$y_i = S(r_i) + \epsilon_i, \quad i = 1, 2, \dots, M. \quad (1b)$$

Here y_i is the measured logarithmic range-normalised backscattered power corresponding to the range r_i . The ϵ_i 's are measurement and process errors associated with the y_i 's whose standard deviations are denoted as σ_i . The σ_i 's, which characterise the uncertainty in the experimental measurement of the y_i 's, are assumed to be known. It is important to remark that these errors may include process contributions arising from multi-scattering effects as well as measurement contributions arising from system sensitivities, non-ideal detector response, and numerical and digitisation errors. Note that eq. (1a) contains two unknown functions, namely $\beta(r)$ and $\alpha_e(r)$. In order to make further progress on the recovery of these functions from the observed data, it is necessary to make an assumption concerning the functional relationship between the backscattering and extinction coefficients. There is experimental

and theoretical evidence [3,6] that under a wide range of atmospheric conditions, these two coefficient functions are approximately related as

$$\beta(r) = \kappa(r)\alpha_0^k(r), \quad (1c)$$

where $\kappa(r)$ is a specified function of range r and the power k for most aerosol clouds of interest is constrained in the interval $0.67 \leq k \leq 1.0$. With this assumed functional dependence between $\beta(r)$ and $\alpha_0(r)$, the lidar inverse problem involves the recovery of the extinction coefficient from a given set of noisy, discrete lidar returns.

7. Now, suppose one is given or has available by an alternative means, some initial (prior) estimate $p(r)$ of the actual, but unknown extinction function $\alpha_0(r)$ ($r \in D$ where D is the support of the extinction function, viz. the set of all points of range r where $\alpha_0(r)$ is non-vanishing). Naturally, this initial estimate is selected to reflect the user's prior information on the extinction function. This prior information may come from initial modelling studies of generic regional aerosols for various atmospheric conditions or from the result of a previous inversion obtained using a different data set. Now, if new information is subsequently obtained about $\alpha_0(r)$ in the form of backscattered return data from some lidar experiment, then, in light of the lidar and measurement models (cf. eqs. (1a), (1b) and (1c)) which relate the extinction coefficient to the measured data, it is possible to specify that the unknown extinction function must be some element of an admissible set Π determined as follows:

$$\Pi = \left\{ q(r) : S(r) = \ln[\kappa(r)] + k \ln[q(r)] - 2 \int_0^r q(r') dr', \sum_{i=1}^M (y_i - S(r_i))^2 / \sigma_i^2 \leq \chi_M^2 \right\}, \quad (2)$$

where χ_M^2 is some prescribed constant that reflects the level of noise corrupting the data. Recall that σ_i is the error in the measurement of the i -th datum. Observe that in the specification of the admissible set, the effect of the noisy observations has been explicitly accounted for by the usual weighted least-squares misfit criterion

$$\chi^2 \equiv \sum_{i=1}^M (y_i - S(r_i))^2 / \sigma_i^2,$$

which is utilised to assess the goodness-of-fit of the theoretical to measured backscattered signals. In view of uncertainties in the measurements, the misfit is specified to be less than or equal to some predetermined constant χ_M^2 . Naturally, if the observation errors constitute an uncorrelated, zero-mean, Gaussian process, then χ^2 possesses a chi-square distribution and this constant can be chosen as $\chi_M^2 \approx M$, the expected value of χ^2 . Actually, this constraint can be relaxed somewhat if one elects to choose instead $\chi_M^2 \leq M + 2\sqrt{2M}$ (i.e., the 95 percent confidence limit for χ^2). However, it is important to emphasize that a statistical model for the observation errors is not required for the ensuing analysis. Indeed, in the absence of an, a priori information on the probabilistic structure of the measurement errors, it is convenient to interpret χ^2 simply as a goodness-of-fit criterion that measures the fidelity of the

model data to the observed data. Then, the selection of $\chi_M^2 = M$ would be interpreted simply as a root-mean-square (RMS) error of 1 in the fit of the model to the observed data.

8. Having specified the precise form of the admissible set Π , suppose now that the initial (prior) estimate $p(r)$ for the extinction function is no longer consistent with the information provided by the observed data, viz. $p(r) \notin \Pi$. Since Π still contains infinitely many extinction functions that are compatible with the observed lidar data, the problem now centers on the choice of the 'best' estimate $q(r)$ of $\alpha_s(r)$ from Π . This choice should not only be based on the given data as embodied in the form of the admissible set Π , but should also incorporate the properties and characteristics of the prior estimate $p(r)$ in some optimal manner. It is proposed that this choice should be based on an information-theoretic concept usually referred to as the principle of minimum cross-entropy. When applied to the present problem, this principle dictates that the final (posterior) estimate $q(r)$ for the extinction coefficient should be chosen as that member of the admissible set Π that possesses the least cross-entropy with respect to the initial (prior) estimate $p(r)$. In other words, the final estimate $q(r)$ is chosen to minimize the cross-entropy between $q(r)$ and $p(r)$ defined as

$$H(q, p) = \int_D q(r) \ln[q(r)/p(r)] dr,$$

subject to the constraint that $q \in \Pi$ (viz., the final estimate should be consistent with all the available information contained in the observed data). It is interesting to note that the principle of minimum cross-entropy is referred to in the literature by a plethora of names depending on the particular field of study and on the context in which it is used, including such terms as discrimination information, I-divergence, directed divergence, relative entropy, Kullback-Leibler information measure and expected weight of evidence [7-12]. Furthermore, it should be noted that this principle can be considered to be the natural generalisation of the maximum entropy (MAXENT) principle which was originally developed by Maxwell, Boltzmann and Gibbs in the context of statistical mechanics and later extended by Jaynes to the status of a general inference procedure of which the statistical mechanics application was merely a special case [13-15]. In particular, the principle of minimum cross-entropy reduces to Jaynes' MAXENT procedure if the prior estimate $p(r)$ is selected to be a constant or uniform function over the domain D .

9. The rationale for the minimum cross-entropy principle as an appropriate criterion of choice is elegantly embodied in the axiomatic formulation of Shore and Johnson [11], who demonstrated that the principle of minimum cross-entropy is the only inference procedure that is consistent with four basic criteria (axioms) that any rational method of inference must possess. These consistency conditions specify, among other things, that the solution provided by any rational inference procedure must be unique and invariant under coordinate transformations. From another viewpoint, the cross-entropy can be interpreted as a measure of the information "distance" between the initial (prior) and final (posterior) estimates $p(r)$ and $q(r)$, respectively. Along this vein, a geometrical interpretation of the minimum cross-entropy principle can be depicted as in Fig. 1. This figure shows the final estimate q for the extinction

function as the information- or I-projection of the initial estimate p onto the admissible set Π . From this perspective, the cross-entropy $H(q, p)$ can be interpreted as the "distance" between the two estimates p and q . Since q is obtained as the I-projection of p onto Π , this distance corresponds to the shortest distance between p and any point of Π , viz.

$$H(q, p) = \min_{q' \in \Pi} H(q', p).$$

10. Hence, $H(q, p)$ can be considered to be a measure of the amount of information encoded in the data that is not already embedded in the initial estimate p . Indeed, the figure indicates that the cross-entropy, when viewed as an information distance between two extinction functions, satisfies a triangle equality of the form

$$H(\alpha_e, p) = H(\alpha_e, q) + H(q, p),$$

a fact that can be readily verified by some simple manipulations. When looked at from this perspective, it is readily apparent that the final estimate q is closer to the true but unknown function α_e than is the initial estimate p since $H(\alpha_e, q) \leq H(\alpha_e, p)$. Indeed, the final estimate q is the estimate closest to the initial estimate p that at the same time verifies the data constraint. Finally, it should be noted that if the data constraints contain no new information on the extinction function other than that already embedded in the initial estimate p , the final estimate q in this case (as selected by the MCE principle) is simply the initial estimate p . This would correspond to the situation where p is a member of the admissible set Π and the I-projection of p on Π would simply be p .

11. In summary then, when provided with an initial (prior) estimate $p(r)$ of the true extinction coefficient $\alpha_e(r)$, the final (posterior) estimate $q(r)$ is chosen, in adherence to the principle of minimum cross-entropy, as that estimate which solves the following functional minimisation problem :

$$\min_{q(r)} \left\{ \int_D q(r) \ln [q(r)/p(r)] dr \right\} \quad (3a)$$

subject to the constraint

$$q(r) \in \Pi, \quad (3b)$$

where Π is the admissible set defined in eq. (2). It is important to note that since Π possesses the geometrical structure of a closed and convex set in the multi-dimensional model space, there exists a unique $q(r) \in \Pi$ (i.e., unique up to a set of measure zero) which solves the minimisation problem posed in eqs. (3a) and (3b) [10]. Additionally, it should also be remarked that the cross-entropy functional $H(q, p)$ is positive and convex in both q and p . Observe at this point that the reconstruction of the extinction coefficient based on the MCE principle requires the solution of a nonlinear functional (i.e., infinite-dimensional) minimisation problem. Unfortunately, there does not exist a closed-form solution for this problem and, consequently, some numerical method must be specified for computing an approximation

to this solution. The development of a numerical algorithm to solve an appropriate finite-dimensional approximation to this problem is the subject of the next section.

NUMERICAL ALGORITHM FOR THE MCE RECONSTRUCTION OF THE EXTINCTION COEFFICIENT

12. Although the MCE reconstruction of $\alpha_e(r)$ has been conceptually formulated as a continuous nonlinear programming problem, it should be emphasized that from a purely computational point of view, it is necessary to discretize the problem and reduce it to the solution of a nonlinear programming problem defined over a finite-dimensional space. First, this requires a finite-dimensional approximation for the extinction function (which is an infinite-dimensional parameter). To this end, the final estimate $q(r)$ is approximated by a piecewise-constant function of the form

$$q(r) = \sum_{i=1}^N q_i \mu_{[r_{i-1}, r_i)}(r),$$

where $\mu_S : D \rightarrow \{0, 1\}$ denotes the characteristic (indicator) function of the set S in the domain D (support of extinction function). Note that $\mu_S(r)$ has value 1 if $r \in S$ and value 0 otherwise. It should be emphasized that q_i in the preceding expression can be interpreted as an estimate for the average of the extinction function over the i -th range cell which is centered at $(r_{i-1} + r_i)/2$. Similarly, a piecewise-constant approximation is adopted for the initial estimate $p(r)$, i.e.

$$p(r) = \sum_{i=1}^N p_i \mu_{[r_{i-1}, r_i)}(r).$$

In writing the preceding expressions, it is tacitly assumed that $r_i > r_{i-1}$ for $i = 1, 2, \dots, N$ and that the sequence of radial knots $\{r_i\}_{i=1}^N$ constitutes a complete partition of the domain D (i.e., of the support of the extinction function).

13. With these approximations, the discrete analog of the cross-entropy functional can be written as

$$H(\vec{q}, \vec{p}) = \sum_{i=1}^N q_i \ln[q_i/p_i] \Delta r_i,$$

where $\vec{q} = (q_1, q_2, \dots, q_N)^T$ and $\vec{p} = (p_1, p_2, \dots, p_N)^T$ are vectors in the Euclidean space of dimension N whose components define the piecewise-constant approximations for $q(r)$ and $p(r)$, respectively. In addition, $\Delta r_i \equiv r_i - r_{i-1}$ ($i = 1, 2, \dots, N$) denotes the i -th incremental interval of the range r . In view of this, the discretized form of the lidar equation (cf. eqs. (1a) and (1c)) reads as

$$\tilde{\mu}_i \equiv S(r_i) = \ln[\kappa(r_i)] + k \ln[q_i] - 2 \sum_{j \leq i} q_j \Delta r_j. \quad (4)$$

Furthermore, the function $\kappa(r)$ as well as k is implicitly assumed to be known from the functional relationship between the backscattering and extinction coefficients. Now, combining all this, leads to

the reformulation of the functional nonlinear minimisation problem of eqs. (3a) and (3b) as the following finite-dimensional nonlinear minimisation problem for the determination of the components of the vector \vec{q} :

$$\min_{\vec{q}} \sum_{i=1}^N q_i \ln[q_i/p_i] \Delta r_i \quad (5a)$$

subject to the constraint

$$\sum_{j=1}^M \frac{(y_j - \hat{y}_j)^2}{\sigma_j^2} \leq \chi_M^2, \quad (5b)$$

where y_j is the j -th observed lidar datum (cf. eq. (1b)) and \hat{y}_j is the j -th model (theoretical) lidar datum computed as per eq. (4).

14. It is convenient at this point to make a few remarks concerning the choice of the number of discrete intervals (N) to use in the piecewise-constant approximation of $q(r)$. Since the optimum range resolution in the direction of the lidar beam path is determined by the pulse duration τ of the laser source, it is sensible to associate the discretisation of $q(r)$ with this quantity. Consequently, since the resolution cell (or scatter cell) has length $c\tau/2$, it is desirable to choose the discretisation so that $\Delta r_i = c\tau/2$ for $i = 1, 2, \dots, N$. With this scheme, there now exists a one-to-one correspondence between the estimate q_i of the extinction function for the i -th range cell and the measured lidar return datum y_i from this cell. Hence, for this discretisation procedure, the number of discrete intervals (N) chosen for the piecewise-constant representation of $q(r)$ is equal to the number of lidar data (M) employed in the inversion.

15. Although no a priori constraints or bounds have been explicitly imposed on the values of the components of the final estimate vector \vec{q} , it is important to remark that, physically, these components must necessarily be non-negative. Strictly speaking, the non-negativity constraint on the solution should be imposed in addition to the data constraint contained in eq. (5b). However, the non-negativity constraint does not need to be explicitly incorporated into the formulation, since it is necessarily taken care of automatically by virtue of the presence of the logarithm in the cross-entropy functional. Now, it should be noted that eqs. (5a) and (5b) poses a convex nonlinear programming problem which can be solved using the ellipsoid algorithm. The ellipsoid algorithm, which gained prominence when Khachian [16] utilized it to demonstrate the existence of a polynomial-time algorithm for the solution of linear programming problems, constitutes a simple and extremely robust procedure for solving convex programming problems such as that presented by eqs. (5a) and (5b). This procedure has the advantage of not requiring the introduction of slack variables and Lagrangian multipliers that has, to date, characterised the numerical solution of all MCE problems [12].

16. The theoretical basis for the ellipsoid algorithm as well as an analysis of its convergence properties can be found in the papers by Shor [17] and by Shor and Gershovich [18] as well as in the survey by Bland, Goldfarb and Todd [19]. A brief description of the algorithm, with particular emphasis on

the details concerning its application to the solution of the MCE reconstruction problem as embodied in eqs. (5a) and (5b) follows. As the name implies, the ellipsoid algorithm begins with the specification of an initial ellipsoid E_0 that is known to contain the optimal solution to the non-linear programming problem and then proceeds to the prescription of a rule for the generation of a sequence of successively smaller ellipsoids (i.e., ellipsoids that possess smaller volumes) each of which is guaranteed to contain the optimal solution. The implementation of the ellipsoid algorithm for the solution of the problem of eqs. (5a) and (5b) consists of the following three steps:

17. *Step 0* : Given the initial (prior) estimate \bar{p} for the extinction function and a target value χ_M^2 for the misfit, supply initial guesses for the final (posterior) estimate \bar{q}^0 of the extinction and for the initial ellipsoid matrix R_0 . With these inputs, it should perhaps be noted that the form of the initial ellipsoid E_0 is determined as

$$E_0 = \left\{ \bar{q} \in R^N : (\bar{q} - \bar{q}^0)^T R_0 (\bar{q} - \bar{q}^0) \leq 1 \right\},$$

where \bar{q}^0 is the center of the ellipsoid and R_0 is the ellipsoid matrix which is necessarily real, symmetric and positive definite. In the absence of any prior information other than that embodied in the initial estimate \bar{p} , it is convenient to choose $\bar{q}^0 = \bar{p}$. If lower and upper bounds can be set on the extinction function, then R_0 can be chosen so that the associated ellipsoid E_0 is the smallest ellipsoid which encloses these bounds. Set the iteration index $k = 0$. Steps 1 and 2 that follow generate a sequence of points \bar{q}^k and matrices R_k that determine a sequence of ellipsoids $E_k \subset R^N$ for $k = 1, 2, \dots$

18. *Step 1* : Compute the misfit χ^2 at the point \bar{q}^k . If χ^2 violates the constraint of eq. (5b) (i.e., $\chi^2 > \chi_M^2$), set $\bar{g} = \nabla_{\bar{q}} \chi^2(\bar{q}^k)$, viz. the gradient of the misfit with respect to the parameter vector \bar{q} evaluated at \bar{q}^k . Otherwise, set $\bar{g} = \nabla_{\bar{q}} H(\bar{q}^k, \bar{p})$, the gradient vector of the cross-entropy with respect to \bar{q} evaluated at \bar{q}^k . If each component of \bar{g} is less than some prescribed tolerance, then terminate the procedure with \bar{q}^k as the optimal point; otherwise, proceed to Step 2.

19. *Step 2* : Compute

$$\bar{d} = -\frac{R_k \bar{g}}{\sqrt{\bar{g}^T R_k \bar{g}}},$$

provided the quantity in the denominator is positive. Otherwise, terminate the procedure with \bar{q}^k as the best point found by the algorithm. The updates for the ellipsoid center \bar{q}^k and the ellipsoid matrix R_k are given by

$$\bar{q}^{k+1} = \bar{q}^k + \bar{d} / (N + 1)$$

and

$$R_{k+1} = \frac{N^2}{N^2 - 1} \left[R_k - \frac{2}{N + 1} \bar{d} \bar{d}^T \right],$$

respectively. Replace k by $k + 1$ and return to Step 1.

20. The flowchart for the ellipsoid algorithm as it applies to the MCE reconstruction of an extinction function is shown in Fig. 2. The algorithm is very simple and can be readily implemented on a computer.

An implementation, which resides as a Microsoft C program, is provided in Appendix B. As a final note, it is important to point out that although the constraint of eq. (5b) restricts the solution to lie on or within some closed hyper-volume in R^N , the optimal point which minimizes the cross-entropy in this region must necessarily lie on the surface of the hyper-volume (viz., on $\chi^2 = \chi_M^2$). The details for the derivation of this result are given in Appendix A. This fact has not been explicitly incorporated in the implementation of the ellipsoid algorithm and, consequently, can be used as an independent check of the optimal solution provided by the algorithm.

SOME MCE INVERSION EXAMPLES

21. The results of some tests of the MCE lidar inversion procedure by means of computer simulations of noisy lidar data are presented in this section. The extinction profiles used in the simulations are composites built up from generic urban, rural and maritime models of aerosol clouds compiled by the Air Force Geophysics Laboratory (AFGL) [20]. The models used in the present study are the same as those used by Bissonnette [21] in his assessment of Klett's analytical inversion of the single-scattering lidar equation. All of the aerosol clouds utilized in this study had a thickness of 1 km and for each prespecified extinction profile, noise-free lidar data $P(r)$ were generated for twenty values of range r spanning the extent of the cloud. Before each of these data sets were inverted by the MCE procedure, they were first corrupted with Gaussian random noise at a prescribed level in order to simulate process and experimental errors. This corrupted data were then logarithmically transformed to form the $S(r)$ data which were used as the input for the inversion procedure.

22. The first simulation model considers a generic maritime aerosol cloud at 90 percent relative humidity with a triangular extinction profile $\alpha_s(r)$ as illustrated by the solid line in Fig. 3. The backscatter coefficient $\beta(r)$ in this example is assumed to be related to $\alpha_s(r)$ according to $\beta(r) = 0.038\alpha_s(r)$. As the first example, twenty simulated lidar data $P(r)$ were generated for this model and these noise-free data were logarithmically transformed to serve as the input for the Klett and the MCE inversion procedure. Klett's exact inversion procedure, with the far-end boundary condition chosen to be the true α_s value of 1.0 km^{-1} , yielded a result that coincided exactly with the true extinction function (cf. Fig. 3). The MCE inversion solution with the prior estimate $p(r)$ of the extinction function chosen to be a constant (uniform) function on the support set D (i.e., $p(r) = 3.8\mu_D(r)$) also gave a result that exactly matched the true extinction profile. For this noise-free data case, it is necessary to set $\sigma_i^2 = 1.0$ for $i = 1, 2, \dots, M$ and to choose the target misfit value χ_M^2 as 0.001. Although both the Klett and the MCE inversion procedures returned the true extinction function under noise-free conditions, it should be emphasized that the MCE solution, unlike the Klett solution, did not require the specification of the correct boundary condition at the far-end of the aerosol cloud. In practical applications, this far-end boundary condition cannot be determined from the given data and would require some additional information concerning the nature of the aerosol cloud. However, it is known [3,21] that the accuracy of extinction profile recovery

ered by the exact far-end lidar inversion procedure depends critically on the proper prescription of this boundary condition. As an example, the dotted-dashed line in Fig. 3 shows a reconstructed extinction function (for the noise-free data set) using Klett's procedure with the far-end boundary condition specified as $\alpha_r = 2.0 \text{ km}^{-1}$. Observe that although the solution converges to the true extinction coefficient at the near-end of the cloud, there is, nevertheless, a moderate error in the determination of the extinction at the far-end of the cloud. As a final point, it should be noted that although the MCE procedure does not require the specification of an appropriate far-end boundary value, it has the disadvantage of being computationally more expensive than the Klett procedure.

23. The next example considers the more realistic case of noisy lidar data. Fig. 4 compares the result of the MCE solution with Klett's analytic solution for the case of lidar data contaminated with 1.5 percent RMS Gaussian noise. For this example, the MCE solution was computed with the prior estimate $p(r)$ chosen to be a constant function and the misfit target value $\chi^2_{M=20}$ set to 20. Note that the MCE solution is smooth and approximates the true extinction profile very well. On the other hand, the Klett solution which was obtained with the correct specification of the far-end boundary value, displayed a moderate amount of oscillation at the far-end of the cloud. Observe that this oscillation is not present at the near-end of the cloud and, in this region, the approximation of the Klett solution to the true extinction profile is as good as that provided by the MCE solution. For even a small level of noise, the MCE inversion procedure provides an intrinsically smoother solution than that given by the Klett inversion procedure. Although both inversion procedures are stable with respect to perturbations in the data, the MCE inversion is not as greatly affected by the noise as is the Klett inversion.

24. Next, some examples which illustrate certain properties of the MCE inversion procedure with respect to the choice of the prior estimate, the choice of the misfit target value, and the level of noise corrupting the lidar data are presented. The result of an MCE inversion for a 3 percent level of RMS Gaussian noise is shown in Fig. 5. The dotted line in Fig. 5 corresponds to an MCE inversion with the prior estimate $p(r)$ chosen to be a constant or uniform function. Again, the misfit target value $\chi^2_{M=20}$ was set to 20. Since the prior estimate is uniform, the MCE inversion result is simply the maximum entropy restoration of the extinction profile and as such, provides the most conservative (uniform) estimate of the extinction consistent with the given data. Observe that the inversion was quite successful since the result $q(r)$ (i.e., the final or posterior estimate of the extinction function) closely approximates the true extinction profile $\alpha_r(r)$, even in the presence of simulated noise. Next, the target misfit value was set to 32 with no changes in the other parameters. The MCE inversion result for this case is displayed by the dashed line in Fig. 5. Since the noise level was slightly overestimated in this example, observe that the resulting inversion is oversmoothed (viz., the peak of the extinction function is underestimated) and, indeed, the final estimate in this case more closely resembles the initial uniform estimate than does the final estimate in the previous example. This should not be too surprising since in the case $\chi^2_M \rightarrow \infty$, the data does not constrain the solution at all and the minimum cross-entropy solution is achieved when

$q(r) = p(r)$, viz. when the final estimate is identical to the initial estimate. This is perfectly natural since one would not desire the final estimate to differ from the initial estimate when the observed data consists of pure noise. It should be remarked that to achieve a good inversion, it is necessary to exercise judgement in the choice of the misfit target value, viz. this value should be chosen so that the solution fits the given data within some expected or prescribed tolerance that adequately reflects the severity of the noise corrupting the data.

25. The second simulation model considered here is constructed from a composite configuration of the generic urban, rural and maritime aerosols at 70 relative humidity with a resulting extinction profile displayed by the solid line in Fig. 6. For this example, the functional dependence between the backscattering and extinction coefficients mirror the regional aerosols used in the construction of the composite model. Hence, the functional dependence is described by a piece-constant relationship of the form

$$\beta(r) = \{0.0096\mu_{0.0.35}(r) + 0.0396\mu_{0.35.0.65}(r) + 0.021\mu_{0.65.1.00}(r)\}\alpha_s(r).$$

Twenty simulated lidar data were generated for this example and corrupted with Gaussian noise at the 5 percent RMS noise level. With the choice of $p(r)$ as a constant or uniform function (i.e., $p(r) = 3.5\mu_D(r)$) and with the misfit target value set at $\chi^2_{M=20} = 20$, the resulting MCE reconstruction of the extinction profile is illustrated by the dotted line of Fig. 6. The resulting inversion is reasonable for the stated noise level. The dashed line of Fig. 6 shows the result of MCE inversion, but this time with an initial estimate $p(r)$ given by

$$p(r) = (1.0 + 10.0r)\mu_{0.0.5}(r) + (11.0 - 10.0r)\mu_{0.5.1.0}(r). \quad (6)$$

Observe that this inversion result is quite similar to the preceding one with the exception that the peak in the extinction profile at $r = 0.5$ km is slightly better defined. This should not be too surprising since the prior estimate $p(r)$ for this case explicitly encoded the presence of this peak and the MCE inversion attempts to choose the final estimate to be as similar to the initial estimate as possible while satisfying the constraints imposed by the data.

26. To provide an idea on the bias and variability in the MCE reconstruction of the extinction function, the MCE inversion procedure was applied fifty times to the same simulated (noiseless) data set (generated for the composite aerosol model described above) to which fifty sets of Gaussian noise have been added at the 5 percent RMS level. For this Monte Carlo simulation, the bias $B(r)$ and variability $V(r)$ in the MCE inversion result is estimated as follows :

$$B(r) = \frac{1}{50} \sum_{i=1}^{50} (q^i(r) - \alpha_s(r))$$

and

$$V(r) = \sqrt{\frac{1}{49} \sum_{i=1}^{50} (q^i(r) - q(r))^2},$$

where $q^i(r)$ denotes the MCE reconstruction of the extinction function for i -th data set and

$$\bar{q}(r) = \frac{1}{50} \sum_{i=1}^{50} q^i(r)$$

is the mean of the 50 MCE inversions. The bias is simply the mean of the fifty differences between the reconstructed and the true extinction functions. The mean $\bar{q}(r)$ of the 50 inversions is shown by the dotted curve in Fig. 7. The error bars show one standard deviations limit (i.e., the variability $V(r)$) computed from the 50 runs. These results indicate that the inversion procedure performs quite adequately especially when one recalls that the data set consists of only twenty noisy lidar measurements. The bias in the recovered extinction profile is on the order of 0.25 km^{-1} whereas the variability is on the order of 0.30 to 0.50 km^{-1} . Observe that the variability in the MCE reconstruction of the extinction coefficient is larger at the far end of the cloud than at the near end. This is due to the fact that the lidar return signal $P(r)$ decreases with range due to attenuation; hence, the corresponding logarithmic range-adjusted signal $S(r)$ exhibits an increasing noise level with increasing range.

27. In actual practice, one never knows the precise relationship between the backscattering and extinction coefficients. To simulate these circumstances, the MCE inversion for the example considered in Fig. 6 is recomputed, but this time with a mis-specification in the functional relationship between the backscattering and extinction coefficients, viz.

$$\beta(r) = \{0.01\mu_{0.035}(r) + 0.025\mu_{0.35,1.0}(r)\}\alpha_e(r).$$

The initial estimate $p(r)$ was chosen as per eq. (6). In order to partially account for the fact that the backscattering-to-extinction ratio is poorly known, the misfit to set value $\chi^2_{L=20}$ was set to 27. The inversion result is shown by the dotted curve in Fig. 8. The similarity between the MCE recovered and the true extinction profile is as good as can be expected especially in light of the incorrect specification of the backscatter-to-extinction ratio and the level of noise in the data. Observe that the main peak in the extinction function at the midpoint of the cloud is well-determined. The inverted result shows the two smaller peaks also, but their values are underestimated.

CONCLUSIONS

28. An information-theoretic approach based on the principle of minimum cross-entropy (MCE) that allows for the rational combination of prior information on the extinction profile with additional information contained in the noisy lidar return data, has been utilized to formulate an inversion procedure for the single-scattering lidar equation. Since non-uniqueness in the inversion of the lidar equation is well-known, the MCE inversion procedure enables the objective reconstruction of a preferred model of the aerosol cloud that not only minimizes extraneous features in the solution not required in order to interpret the data, but also optimally accounts for errors in the data. Indeed, the nature of the model

recovered depends as much on the data errors as on the data themselves. A good inversion result, which does not mislead the user, can only be obtained if all available information on the problem is effectively utilised and this, of course, includes the information on the level of noise corrupting the data.

29. A simple and robust numerical procedure based on the ellipsoid algorithm has been developed for the solution of the highly nonlinear, constrained minimisation problem required to perform the MCE reconstruction. Numerical results obtained using simulated noisy lidar data show that the algorithm for MCE reconstruction provides stable and well-behaved estimates for the extinction function. In addition, this numerical procedure seems to be quite insensitive to the given initial guess for the final estimate $q(r)$. Except for the initial phase, the ellipsoid algorithm generally exhibited convergence to the optimal MCE solution at a linear rate, a result that has been predicted by theory [17]. Consequently, while the algorithm displays quite a rapid convergence to the near-optimal solution after the first twenty to forty iterations, it nevertheless exhibits a slow convergence rate near the optimal solution. Since it takes from about 30 to 60 seconds to compute an MCE inversion solution on an IBM PC-XT, the algorithm in its present form is not suitable for a real-time recovery of extinction profiles, unless either an algorithm with a better convergence rate near the optimal solution is developed or a computer with a faster processor is used.

REFERENCES

1. Evans, B.T.N., "Field Evaluations of a Canadian Laser Cloud Mapper and Candidate IR Screening Aerosols", Smoke/Obscurants Symposium VI, Unclassified Section, Harry Diamond Labs, Adelphi, Md., April 1982.
2. Hitchfeld, W. and Bordan, J., "Errors Inherent in the Radar Measurement of Rainfall at Attenuating Wavelengths", *J. of Meteorology*, Vol. 11, pp. 58-67, 1954.
3. Klett, J.D., "Stable Analytical Inversion Solution for Processing Lidar Returns", *Applied Optics*, Vol. 20, pp. 211-220, 1981.
4. Evans, B.T.N., "On the Inversion of the Lidar Equation (U)", DREV R-4343/84, November 1984, UNCLASSIFIED
5. Kohl, R.H., "Discussion of the Interpretation Problem Encountered in Single-Wavelength Lidar Transmissometers", *J. of Applied Meteorology*, Vol. 17, pp. 1034-1038, 1978.
6. DeLeeuw, G., "Mie Scattering on Particle Size Distributions: Influence of Size Limits and Complex Refractive Index on the Calculated Extinction and Backscatter Coefficients", Physics Laboratory TNO, PHL 1982-50, The Netherlands, 1982.
7. Kullback, S., *Information Theory and Statistics*, New York: Dover, 1969.
8. Pinsker, M.S., *Information and Information Stability of Random Variables and Processes*, San Francisco, CA: Holden-Day, 1964.
9. Good, I.J., *Probability and the Weighting of Evidence*, London, England: Griffen, 1950.
10. Csizsar, I., "I-divergence Geometry of Probability Distributions and Minimisation Problems", *Ann. Prob.*, Vol. 3, pp. 146-158, 1975.
11. Shore, J.E. and Johnson, R.W., "Axiomatic Derivation of the Principle of Maximum Entropy and the Principle of Minimum Cross-Entropy", *IEEE Trans. Inform. Theory*, Vol. IT-26, pp. 26-37, 1980.
12. Shore, J.E. and Johnson, R.W., "Properties of Cross-Entropy Minimisation", *IEEE Trans. Inform. Theory*, Vol. IT-27, pp. 472-482, 1981.
13. Elsascer, W.M., "On Quantum Measurements and the Role of the Uncertainty Relations in Statistical Mechanics", *Phys. Rev.*, Vol. 52, pp. 978-999, 1937.

14. Jaynes, E.T., "Information Theory and Statistical Mechanics I", Phys. Rev., Vol. 108, pp. 171-190, 1957.
15. Jaynes, E.T., "Prior Probabilities", IEEE Trans. Syst. Sci. Cybern., Vol. SSC-4, pp. 227-241, 1968.
16. Khachian, L.G., "A Polynomial Algorithm in Linear Programming", Soviet Mathematics Doklady, Vol. 20, pp. 191-194, 1979.
17. Shor, N.Z., "Cut-off Method With Space Extension in Convex Programming Problems", Cybernetics, Vol. 12, pp. 94-96, 1977.
18. Shor, N.Z. and Gershovich, V.I., "Family of Algorithms for Solving Convex Programming Problems", Cybernetics, Vol. 15, pp. 502-508, 1980.
19. Bland, R.G., Goldfarb, D. and Todd, M.J., "The Ellipsoid Method: A Survey", Operations Research, Vol. 29, pp. 1039-1091, 1981.
20. Shettle, E.P. and Fain, R.W., "Models for the Aerosols of the Lower Atmosphere and the Effects of Humidity Variations on Their Optical Properties (U)", AFGL-TR-79-0214, Air Force Geophysics Laboratory, Hanscom AFB, Massachusetts, September 1979, UNCLASSIFIED
21. Bissonnette, L.R., "Multiscattering Lidar Method for Determining Optical Parameters of Aerosols (U)", DREV R-4430/86, October 1986, UNCLASSIFIED

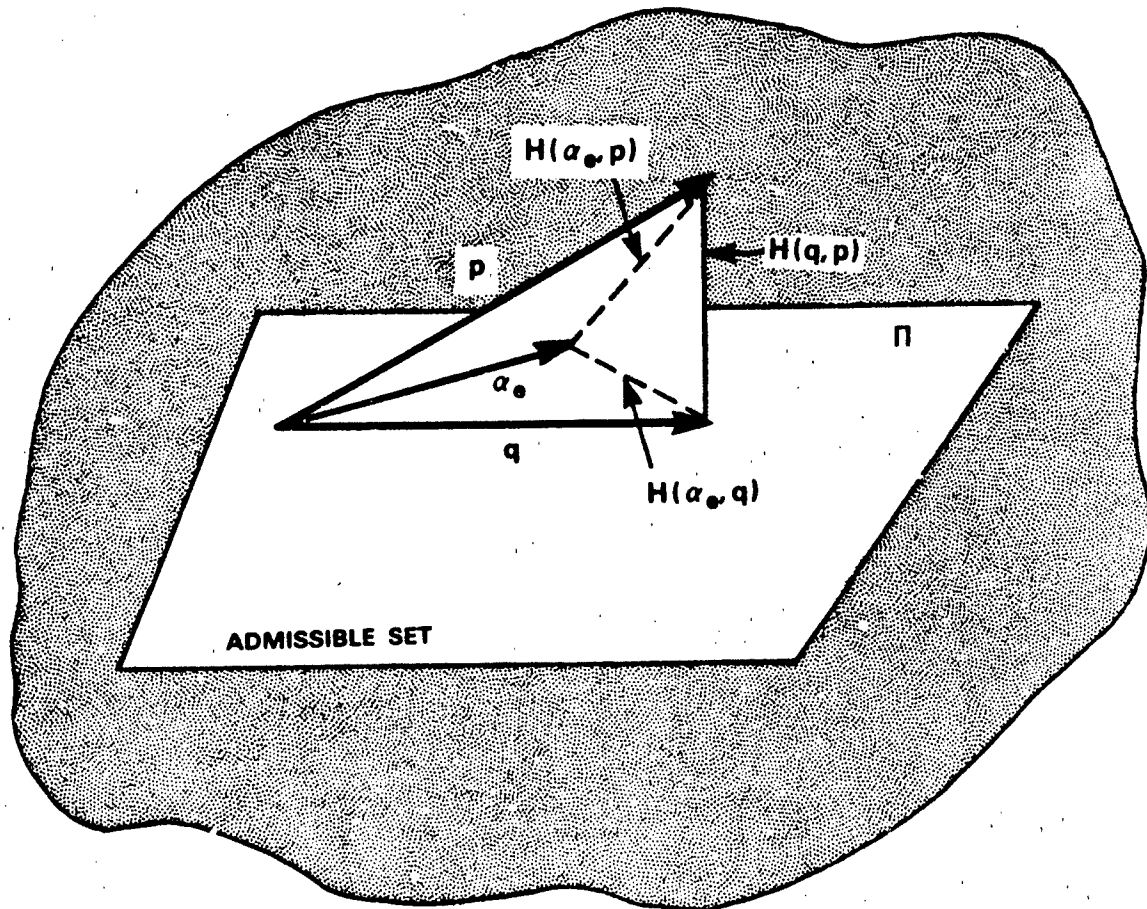


Figure 1

Schematic illustration of the geometric interpretation of the principle of minimum cross-entropy. Here, the final estimate q is the information- or I-projection of the initial estimate p onto the admissible set Π . Consequently, q is as close to p as possible in the information measure sense while at the same time satisfying the new information encoded in the data.

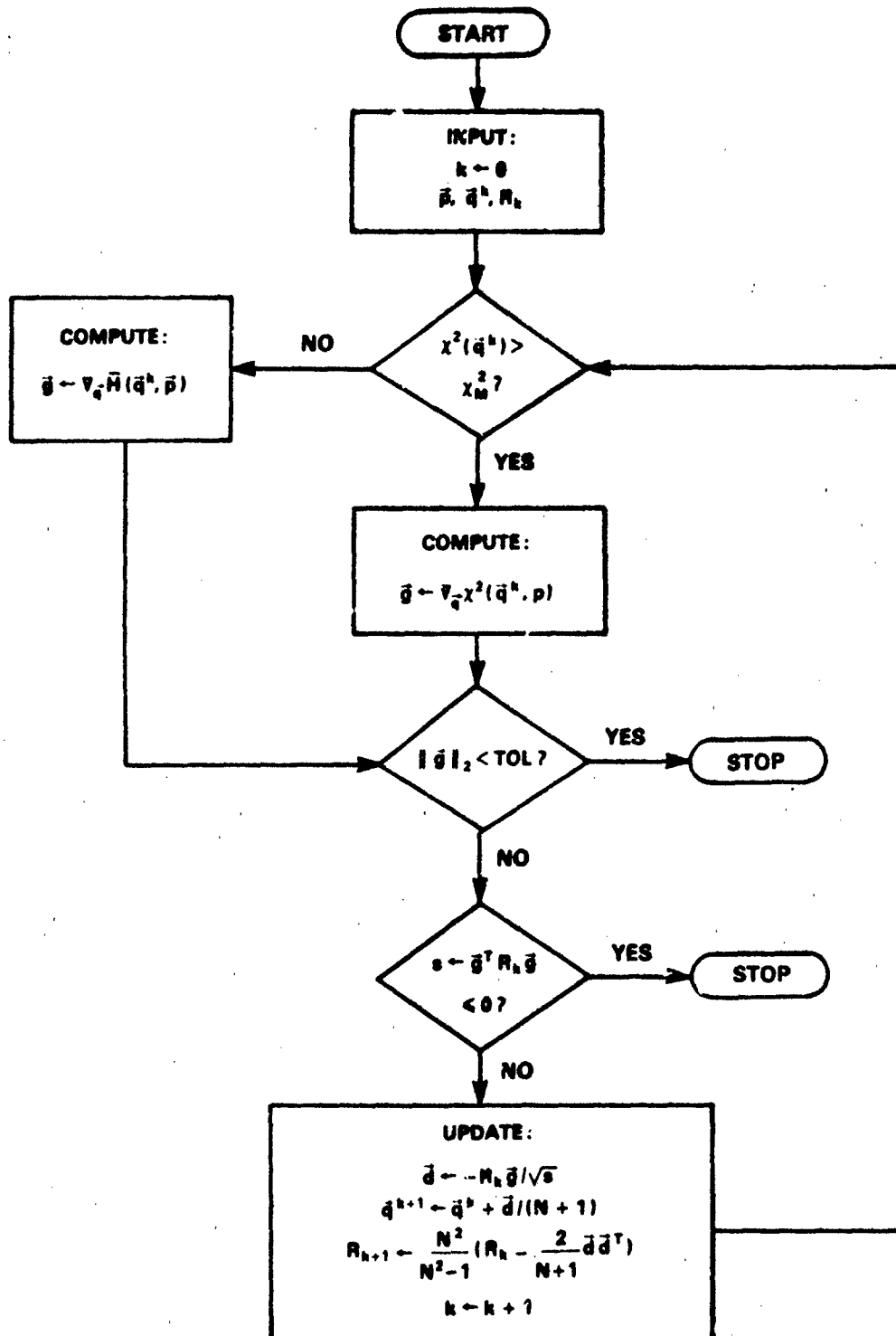


Figure 2

Flowchart of the ellipsoid algorithm used to solve the convex optimization problem required to obtain the minimum cross-entropy recovery of the extinction function.

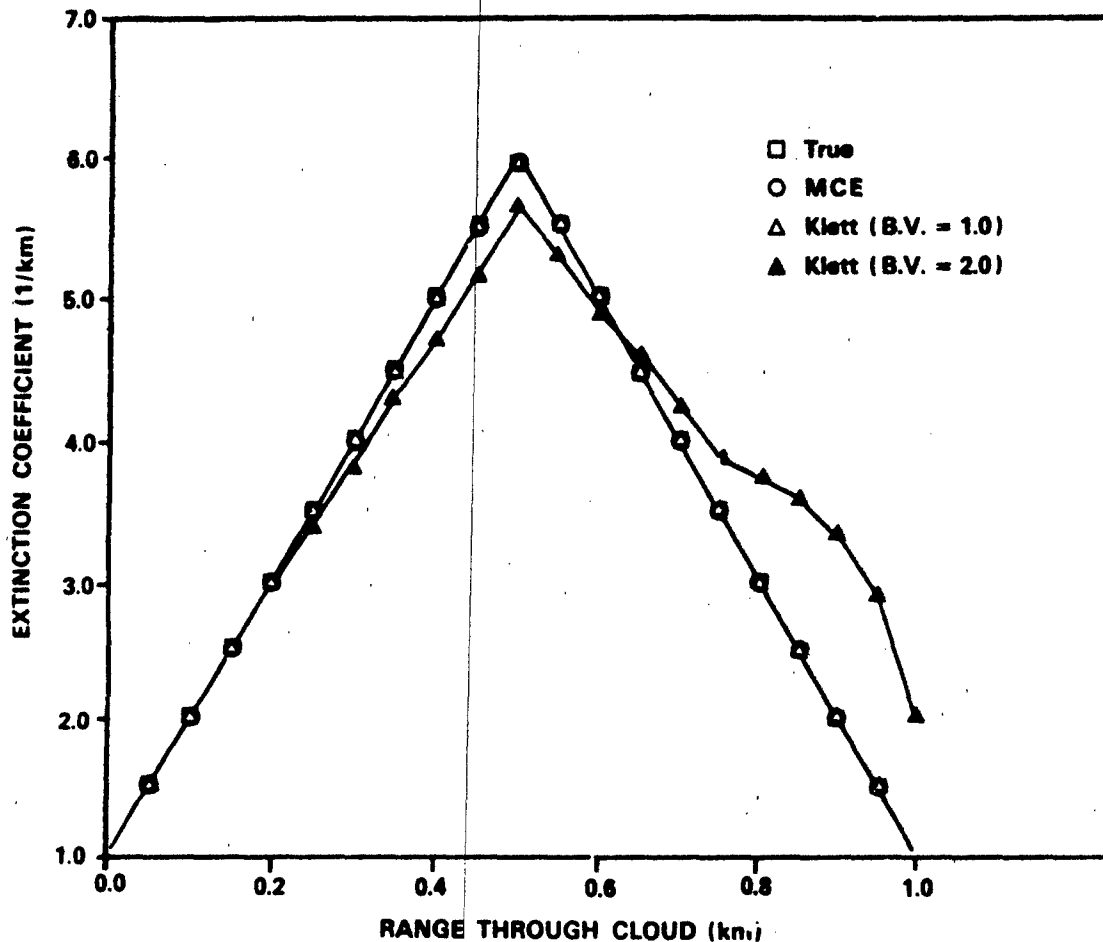


Figure 3

A comparison of the aerosol extinction functions recovered by the MCE and Klett inversion procedures for the case of noise-free lidar data. The true extinction function (solid line) is representative of a generic maritime aerosol cloud at 99% relative humidity. The MCE solution and the Klett solution, with a correctly specified far-end boundary condition, returned extinction profiles that coincided with the true extinction coefficient. The dashed line shows the Klett solution with an incorrectly specified far-end boundary value on the aerosol cloud.

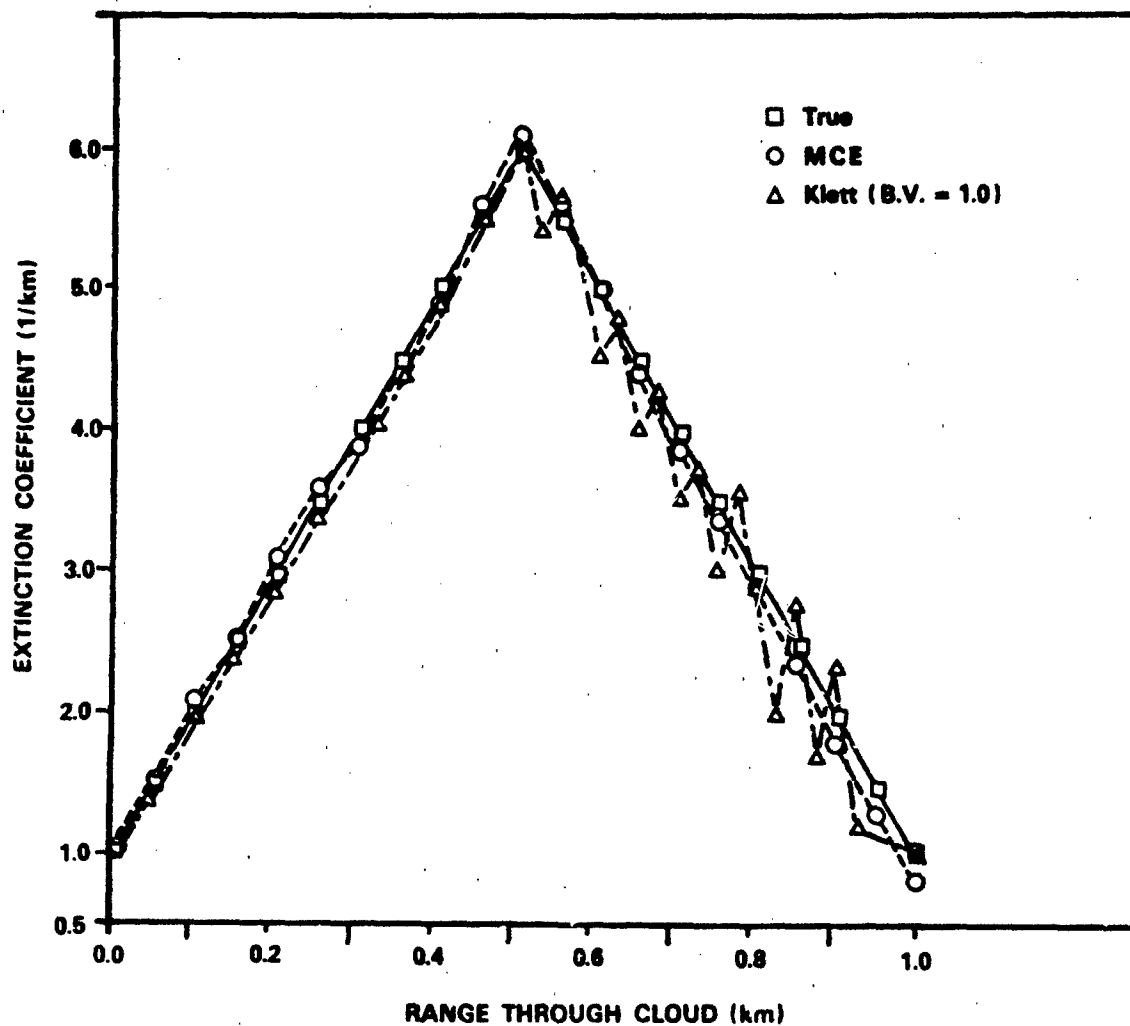


Figure 4

A comparison of the aerosol extinction functions recovered by the MCE and Klett inversion procedures for the case of lidar data contaminated with 1.5% RMS Gaussian noise. The Klett solution was obtained with the correctly specified far-end boundary value for the aerosol cloud. Note that the MCE solution is much smoother than the Klett solution.

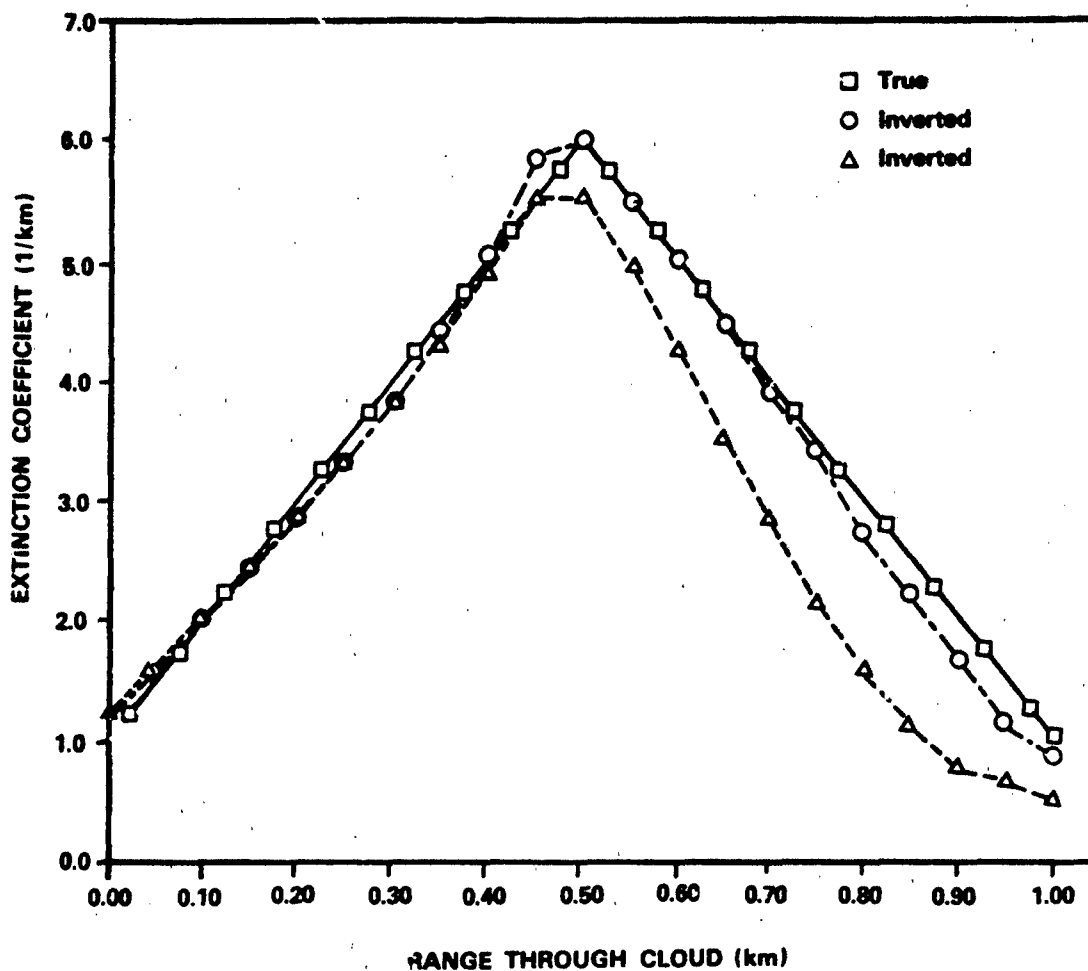


Figure 5

An example of minimum cross-entropy recovered aerosol extinction functions. The dotted and dashed lines indicate MCE inversions with target misfit values of 20 and 32, respectively. The input for these inversions consisted of 20 lidar data degraded with 3% RMS Gaussian noise.

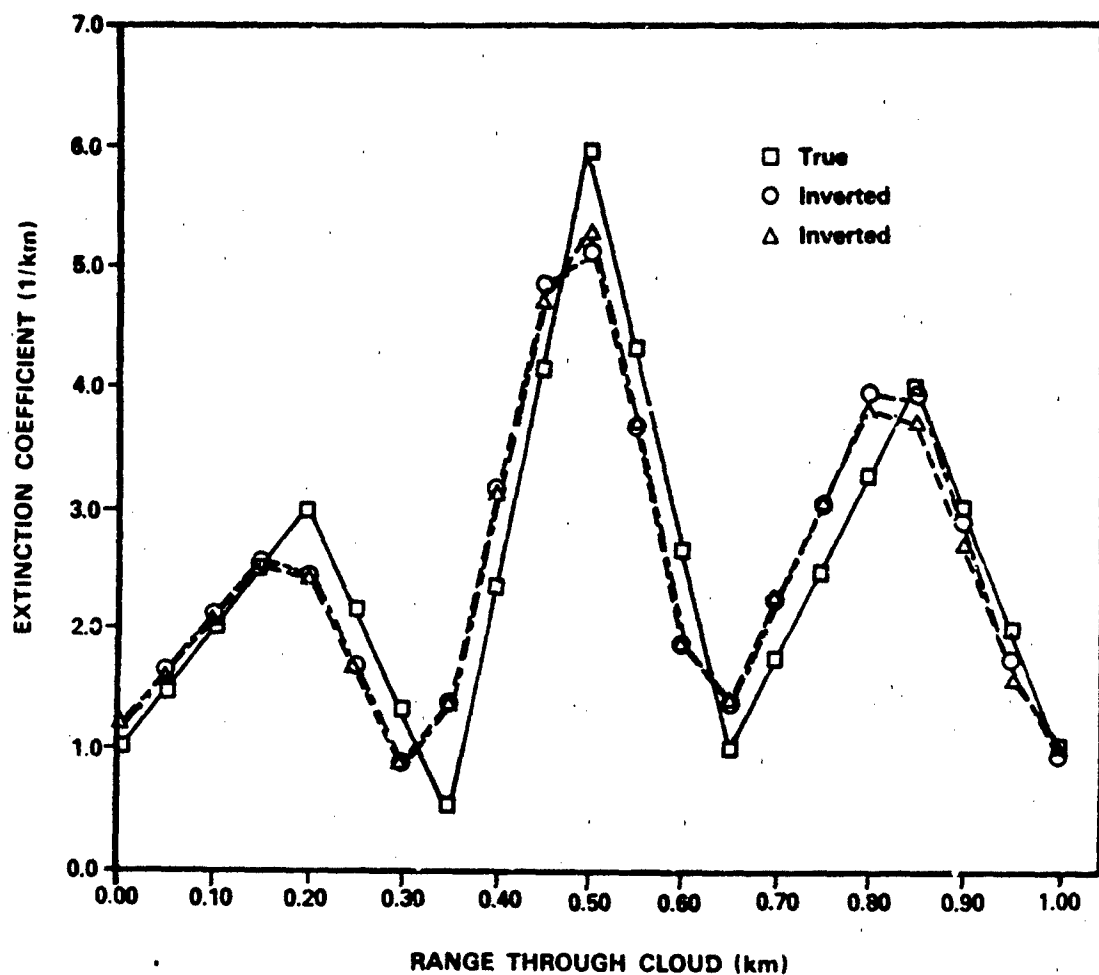


Figure 6

Another example of minimum cross-entropy recovered aerosol extinction functions. The true extinction function (solid line) is a composite constructed by piecing together generic aerosol cloud characteristics from urban, rural and maritime environments. The dotted line indicates the inverted extinction function for a target misfit value of 20 and a uniform initial estimate. The dashed line shows the inverted extinction function using a target misfit value of 20 and the initial estimate as the triangular extinction profile shown by the solid line in Figure 5. These inversions were obtained from twenty lidar data which have been corrupted with 5% RMS Gaussian noise.

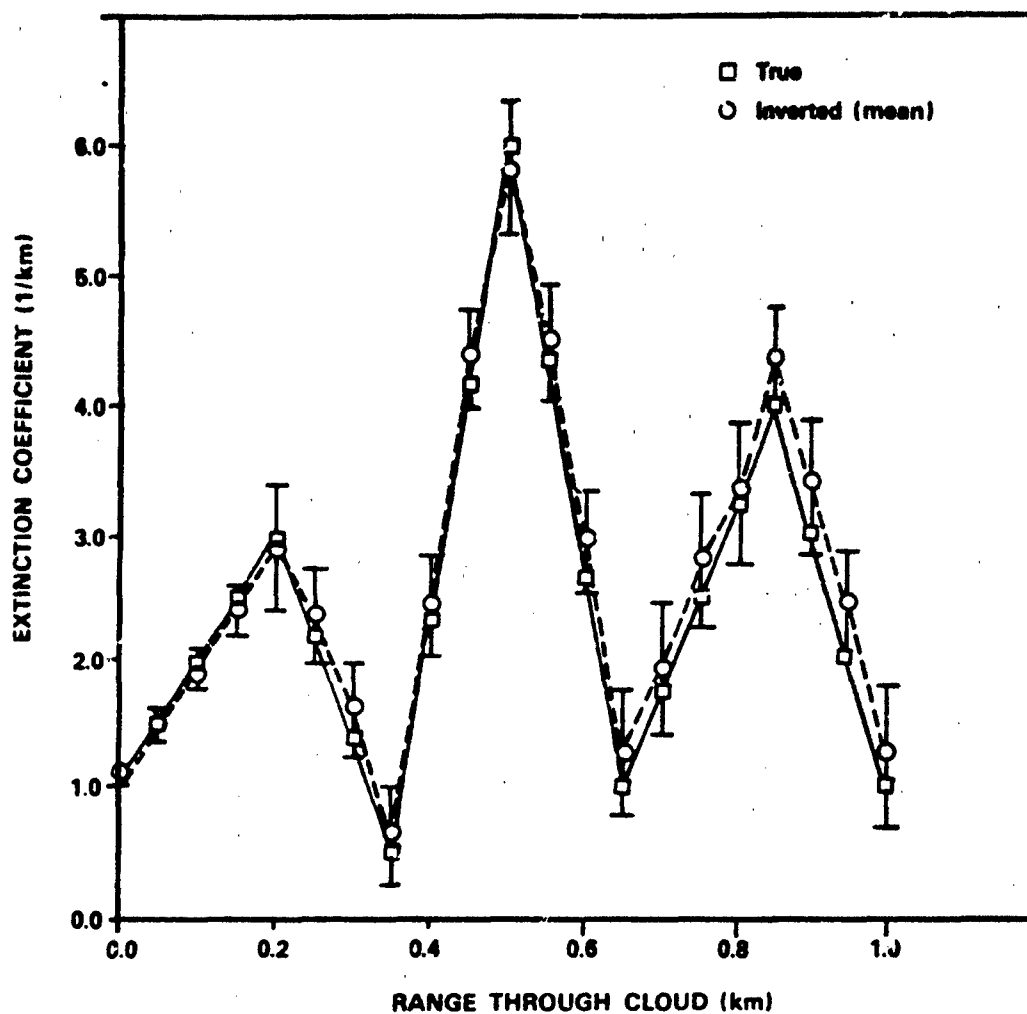


Figure 7

Same example as in Figure 6 but the dotted line shows the means of 50 MCE inversions. The error bars indicate the standard deviations for the inversions computed from the fifty noise-corrupted lidar data sets. Each data set consisted of 20 data that has been corrupted with 5% RMS Gaussian noise.

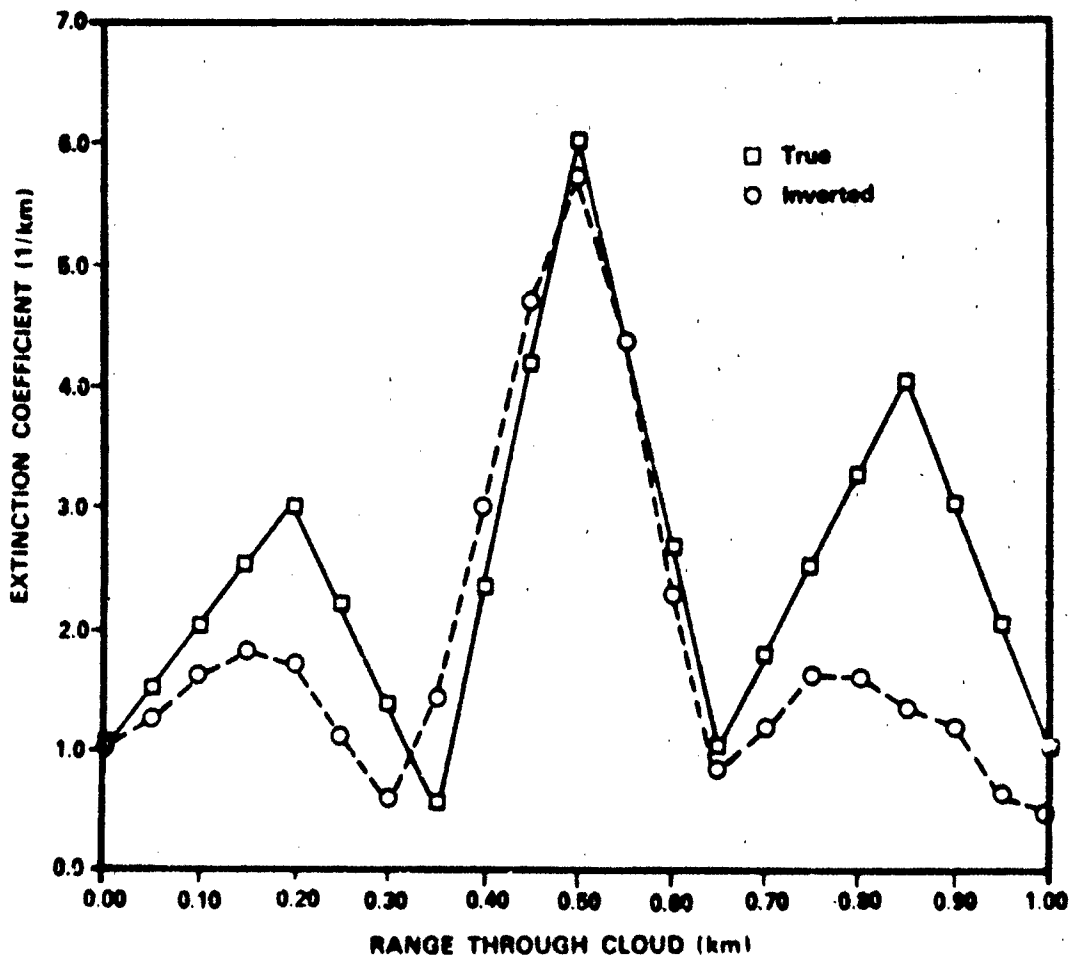


Figure 8

Same example as in Figure 6 but the dotted line shows the recovered extinction profile for a misspecified backscattering-to-extinction ratio. The target misfit value was set to 27 for the inversion.

UNCLASSIFIED

APPENDIX A

30. The MCE reconstruction of the extinction coefficient of aerosol clouds from incomplete and uncertain lidar return data is formulated as the problem of finding the particular solution that possesses the smallest cross-entropy relative to some initial (prior) model and that at the same time provides a satisfactory match between the model (theoretical) and actual data, with this match being quantified by a data misfit of the form

$$\chi^2 = \sum_{i=1}^M \frac{(y_i - S(r_i))^2}{\sigma_i^2} \leq \chi_M^2. \quad (A.1)$$

It will be shown that the solution of minimum cross-entropy lies on the boundary of the allowable misfit region defined above (viz., on the surface $\chi^2 = \chi_M^2$) rather than in its interior. In other words, the solution of minimum cross-entropy is necessarily the one that corresponds to the largest permissible misfit.

31. This result is easily demonstrated by contradiction. Suppose that the minimum cross-entropy solution $q^*(r)$ results in the data misfit $\chi^{*2} < \chi_M^2$, i.e. the solution lies in the interior of the misfit region. Now consider a solution given by $q(r) = \nu q^*(r)$ ($\nu \in (0, 1)$) with the corresponding data misfit χ^2 . Next, express χ^2 in terms of χ^{*2} by substituting $q(r)$ into eq. (A.1) and performing some simple algebraic manipulations to obtain

$$\chi^2 = \chi^{*2} + (1 - \nu)c_1(q^*(r)) + (1 - \nu^2)c_2(q^*(r)) + (\ln \nu)c_3(q^*(r)) + k^2 \ln^2(\nu), \quad (A.2)$$

where c_1 , c_2 , and c_3 are constants that depend on $q^*(r)$. Now, observe from eq. (A.2) that if ν is selected to be sufficiently close to one, it is possible for χ^2 to be strictly less than χ_M^2 , since $\chi^{*2} \leq \chi_M^2$ by supposition (viz., the point $q(r)$ lies strictly in the interior of the acceptable misfit region). However, note that the value of the cross-entropy functional at $q(r)$ is less than the assumed minimum value of the functional at $q^*(r)$ since

$$\begin{aligned} H(q, p) &= \nu \int_D q^*(r) \ln [q^*(r)/p(r)] dr + \nu \ln \nu \int_D q^*(r) dr \\ &= \nu H(q^*, p) + \nu \ln \nu \int_D q^*(r) dr. \end{aligned}$$

Consequently, in contradiction with the original hypothesis, the MCE solution must lie on the boundary of the allowable misfit region.

UNCLASSIFIED

APPENDIX B

32. The program given in this Appendix is an implementation of the ellipsoid algorithm for the MCE reconstruction of extinction functions for aerosol clouds. It is written in Microsoft C which conforms to the forthcoming American National Standards Institute (ANSI) standard for the C language. It should be emphasized that the program has been written largely for clarity rather than speed and, as such, should not be regarded as a production software implementation of the algorithm. It is primarily intended for research purposes.

UNCLASSIFIED

```
#include <stdio.h>
#include <math.h>
```

```

/*****
/*
/*-----
/*
/* Title : An Information-Theoretic Method for the Inversion of
/*         the Lidar Equation
/*
/* Date: October, 1987
/*
/* Author: Eugene Yee
/*         Defence Research Establishment Suffield
/*         Ralston, Alberta          TOJ 2N0
/*
/*-----
/*
/* Name - ellipsoid(N,N_DATA,eps,chi_sqr,max_iter,data,
/*              variance,R_pos,vec_r,delta_r,x,y,rmatrix)
/*
/* Purpose - To apply the ellipsoid algorithm to solve the
/*            nonlinear programming problem required to compute
/*            the minimum cross-entropy reconstruction of the
/*            extinction coefficient functions.
/*
/*-----
/* Description of parameters in the argument list:
/*
/*      N          number of variables
/*      N_DATA     number of data points
/*      eps        accuracy required in each element of gradient
/*                vector
/*      chi_sqr    target value for the misfit
/*      max_iter   maximum allowable number of iterations of
/*                ellipsoid algorithm
/*      data       array that stores the lidar return data
/*      variance   variance in the values in the data[] array
/*      R_pos      array storing radial values r at which lidar
/*                data stored in data[] are measured
/*      vec_r      array storing radial values r at which
/*                extinction function will be estimated
/*      delta_r    array that stores incremental radial values
/*                between elements specified in array vec_r[]
/*      x          stores current approximation of extinction
/*                function. An initial approximation should be
/*                provided on entry, which will be replaced by
/*                the MCE estimate on exit.
/*      y          stores the initial (prior) estimate of the
/*                extinction function
/*      rmatrix    used to store the current ellipsoid matrix
/*                generated. An initial ellipsoid matrix should
/*                be provided on entry.
/*
/*-----
/* Remarks:
/*
/* 1) The user must provide two functions :

```

[illegible]

```

)
/* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>> */
/* Now begin the computational work of STEP 1.          */
/* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>> */

chi_sqr = 0.0;
for(anInt = 0; anInt < N_DATA; anInt++)
(
    Rj=R_pos[anInt];
    tmp=data[anInt]-model_data(N,Rj,vec_r,delta_r,x);
    tmp1=tmp*tmp;
    chi_sqr += tmp1/variance[anInt];
)

g1 = chi_sqr - chi_sqrm;
if(g1 > eps)                      /* Is constraint violated ? */
(
/* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>> */
/* Calculate the gradient of the constraint           */
/* function.                                           */
/* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>> */

    for(anInt = 0; anInt < N; anInt++)
        grad[anInt]=0.0;   /* Zero gradient array. */
    for(anInt = 0; anInt < N_DATA; anInt++)
    (
        Rj=R_pos[anInt];
        grad_dj(N,Rj,vec_r,delta_r,x,gnd dj);
        tmp = data[anInt]-model_data(N,Rj,vec_r,delta_r,x);
        tmp1 = tmp/variance[anInt];
        for(anInt1 = 0; anInt1 < N; anInt1++)
            grad[anInt1] -= 2.0*tmp1*gnd_dj[anInt1];
    )
)
else
(
/* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>> */
/* Calculate the gradient of the objective             */
/* function (i.e., the cross-entropy function)         */
/* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>> */

    for(anInt = 0; anInt < N; anInt++)
        grad[anInt] = (1.0+log(x[anInt]/y[anInt]));
)

/* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>> */
/* Test for convergence                               */
/* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>> */

for(anInt = 0; anInt < N; anInt++)
    if(grad[anInt] > eps) goto step_two;
return;      /* Convergence of algorithm: exit */

/* ----- */
step two:
```



```

/* ***** */
/* Normalize the gradient array grad[] by its length. The Euclidean length of grad[] is computed using the function euclid_norm(). ***** */
tmp = euclid_norm(N,grad);
for(anInt = 0; anInt < N; anInt++)
    grad[anInt] /= tmp;

/* ***** */
/* In this first portion of STEP 2, we make all the needed calculations for the update of the parameter vector (which determines the center of the ellipsoid) and the ellipsoid matrix rmatrix[][]. ***** */
for(anInt = 0; anInt < N; anInt++)
(
    tmparr[anInt]=0.0;
    for(anInt1 = 0; anInt1 < N; anInt1++)
        tmparr[anInt] += rmatrix[anInt][anInt1]*grad[anInt1];
)
tmp=0.0;
for(anInt = 0; anInt < N; anInt++)
    tmp += grad[anInt]*tmparr[anInt];
if(tmp <= eps)
(
    printf(" Ellipsoid no longer positive definite.\n");
    return; /* exit */
)
tmp1 = sqrt(tmp);

for(anInt = 0; anInt < N; anInt++)
    tmparr[anInt] = -tmparr[anInt]/tmp1;

/* ***** */
/* After these preliminary computations, we are now ready to update the paramter vector x[] and the ellipsoid matrix rmatrix[][]. ***** */

tmp = 2.0/NP_ONE;
for(anInt = 0; anInt < N; anInt++)
( /* First update parameter vector. */
    x[anInt] += tmparr[anInt]/NP_ONE;
    for(anInt1 = 0; anInt1 < N; anInt1++)
    ( /* Next update the ellipsoid matrix. */
        rmatrix[anInt][anInt1] -=
            tmp*tmparr[anInt]*tmparr[anInt1];
        rmatrix[anInt][anInt1] *= FACTOR;
    )
)
)

/* ***** */

```

```

/* After these updates, go back to STEP 1 and */
/* start all over again. */
/* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>> */

goto step_one;
)

/*****
/* Name - grad_dj(N,Rj,vec_r,delta_r,x,gnd_dj)
/* Purpose - To compute the gradient of the j-th model data
/* point d[j] with respect to the parameter vector
/* x = (q[1],...,q[N]) with the result placed in the
/* gnd_dj[]. Rj is the range corresponding to the
/* model datum.
*****/

grad_dj(N,Rj,vec_r,delta_r,x,gnd_dj)

unsigned int N;
double Rj,vec_r[],delta_r[],x[],gnd_dj[];
{
    register int anInt;
    double K;

    K = 1.0;
    for(anInt = 0; anInt < N; anInt++)
    {
        if((anInt == 0) && (vec_r[anInt] > Rj))
            gnd_dj[anInt] = -2.0*delta_r[anInt] + K/x[anInt];
        else if(vec_r[anInt] < Rj)
            gnd_dj[anInt] = -2.0*delta_r[anInt];
        else if(vec_r[anInt-1] <= Rj && Rj < vec_r[anInt])
            gnd_dj[anInt] = -2.0*delta_r[anInt] + K/x[anInt];
        else gnd_dj[anInt] = 0.0;
    }
}

/*****
/* Name - model_data(N,Rj,vec_r,delta_r,x)
/* Purpose - To compute the j-th component of the model data
/* vector corresponding to range Rj.
*****/

double model_data(N,Rj,vec_r,delta_r,x)

unsigned int N;
double Rj,vec_r[],delta_r[],x[];
{
    register int anInt;
    double tmp,CKST1,CKST2,CKST3,K;

    tmp=0.0;
    CKST1 = 0.01;

```

```

CKST2 = 0.025;
CKST3 = 0.025;
K = 1.0;

```

```

for anInt = 0; anInt < N; anInt++)
(
    if((anInt == 0) && (vec_r[anInt] > Rj))
        tmp += x[anInt]*delta_r[anInt];
    else if(vec_r[anInt] <= Rj)
        tmp += x[anInt]*delta_r[anInt];
    else if((vec_r[anInt-1] < Rj) && (Rj < vec_r[anInt]))
        tmp += x[anInt]*delta_r[anInt];
    else
        continue;
)

for(anInt = 0; anInt < N; anInt++)
(
    if(vec_r[anInt] >= Rj && Rj <= 0.35)
        return(log(CKST1)+K*log(x[anInt])-2.0*tmp);
    else if(vec_r[anInt] >= Rj && Rj > 0.35 && Rj <= 0.65)
        return(log(CKST2)+K*log(x[anInt])-2.0*tmp);
    else if(vec_r[anInt] >= Rj && Rj > 0.65 && Rj <= 1.0)
        return(log(CKST3)+K*log(x[anInt])-2.0*tmp);
)
)

```

```

/*****
/*
/*      Name : euclid_norm(num_elem,array)
/*
/*      Purpose:  Compute the euclidean norm of vector array[] of
/*                  num_elem.
/*
/*
*****/

```

```

double euclid_norm(num_elem,array)

int num_elem;
double array[];
{
    register int anInt;
    double norm;

    norm = 0.0;

    for(anInt = 0; anInt < num_elem; anInt++)
        norm += array[anInt]*array[anInt];

    return(sqrt(norm));
}

```

DOCUMENT CONTROL DATA - R & D		
(Security classification of title, body of abstract and indexing annotation must be entered when the overall document is classified)		
1. ORIGINATING ACTIVITY		2a. DOCUMENT SECURITY CLASSIFICATION
DEFENCE RESEARCH ESTABLISHMENT SUFFIELD		UNCLASSIFIED
		2b. GROUP
3. DOCUMENT TITLE		
IN INFORMATION-THEORETIC METHOD FOR THE INVERSION OF THE LIDAR EQUATION		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)		
SUFFIELD MEMORANDUM		
5. AUTHOR(S) (Last name, first name, middle initial)		
YEE, EUGENE		
6. DOCUMENT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
May 1988		21
8a. PROJECT OR GRANT NO.	8b. ORIGINATOR'S DOCUMENT NUMBER(S)	
	1225	
8c. CONTRACT NO.	8d. OTHER DOCUMENT NO.(S) (Any other numbers that may be assigned this document)	
10. DISTRIBUTION STATEMENT		
UNLIMITED		
11. SUPPLEMENTARY NOTES		12. SPONSORING ACTIVITY
13. ABSTRACT		
<p>A new solution is presented for the reconstruction of profiles of aerosol volume extinction coefficients from the noisy backscattered returns of a monostatic single-wavelength lidar system. This inverse problem is solved by utilizing an information-theoretic method based on the principle of minimum cross-entropy (MCE), which represents an objective and rational approach for the effective incorporation, into the inversion procedure, of both prior information in the form of an initial estimate of the extinction coefficient and additional information in the form of the observed data. A simple and robust numerical procedure, based on the ellipsoid algorithm, is developed to compute the MCE reconstruction of the extinction function. A number of numerical examples, based on noisy synthetic lidar data, are employed to demonstrate and evaluate the utility and efficacy of the inversion method.</p>		

UNCLASSIFIED

This Sheet Security Classification

KEY WORDS

Lidar
Inversion
Information Theory
Cross-entropy
Ellipsoid Algorithm

INSTRUCTIONS

1. **ORIGINATING ACTIVITY** Enter the name and address of the organization issuing the document.
- 2a. **DOCUMENT SECURITY CLASSIFICATION** Enter the overall security classification of the document including special warning terms whenever applicable.
- 2b. **GROUP** Enter security reclassification group number. The three groups are defined in Appendix "M" of the DRB Security Regulations.
3. **DOCUMENT TITLE** Enter the complete document title in all capital letters. Titles in all cases should be unclassified. If a sufficiently descriptive title cannot be selected without classification, show title classification with the usual one capital-letter abbreviation in parentheses immediately following the title.
4. **DESCRIPTIVE NOTES** Enter the category of document, e.g. technical report, technical note or technical letter. If appropriate, enter the type of document, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.
5. **AUTHOR(S)** Enter the name(s) of author(s) as shown on or in the document. Enter last name, first name, middle initial. If military, show rank. The name of the principal author is an absolute minimum requirement.
6. **DOCUMENT DATE** Enter the date (month, year) of Establishment approval for publication of the document.
- 7a. **TOTAL NUMBER OF PAGES** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.
- 7b. **NUMBER OF REFERENCES** Enter the total number of references cited in the document.
- 8a. **PROJECT OR GRANT NUMBER** If appropriate, enter the applicable research and development project or grant number under which the document was written.
- 8b. **CONTRACT NUMBER** If appropriate, enter the applicable number under which the document was written.
- 9a. **ORIGINATOR'S DOCUMENT NUMBER(S)** Enter the official document number by which the document will be identified and controlled by the originating activity. This number must be unique to this document.
- 9b. **OTHER DOCUMENT NUMBER(S)** If the document has been assigned any other document numbers (either by the originator or by the sponsor), also enter this number(s).
10. **DISTRIBUTION STATEMENT** Enter any limitations on further dissemination of the document, other than those imposed by security classification, using standard statements such as:
 - (1) "Qualified requesters may obtain copies of this document from their defense documentation center."
 - (2) "Announcement and dissemination of this document is not authorized without prior approval from originating activity."
11. **SUPPLEMENTARY NOTES** Use for additional explanatory notes.
12. **SPONSORING ACTIVITY** Enter the name of the departmental project office or laboratory sponsoring the research and development. Include address.
13. **ABSTRACT** Enter an abstract giving a brief and factual summary of the document, even though it may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall end with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (TS), (SI), (C), (RI), or (U).

The length of the abstract should be limited to 20 single-spaced standard typewritten lines, 7 1/2 inches long.
14. **KEY WORDS** Key words are technically meaningful terms or short phrases that characterize a document and could be helpful in cataloging the document. Key words should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words, but will be followed by an indication of technical context.